



Parametric analysis of heat and mass transfer regenerators using a generalized effectiveness-NTU method

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ABSTRACT

A parametric analysis of heat and mass transfer in regenerative exchangers that employ sorbent materials has been performed. The adopted methodology is based on the effectiveness-NTU (number of transfer units) approach, traditionally employed for analyzing sensible heat exchangers. However, a generalization was developed to account for the effects of coupled heat and mass transfer, matrix diffusion, as well as physical adsorption. The generalized approach was applied to a unified mathematical formulation for this class of exchangers and the potential of the methodology was demonstrated through an analysis of the influence of several dimensionless parameters on regenerator performance.

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1. Introduction

The simulation of coupled heat and mass transfer in regenerators with adsorptive materials has become an essential tool for obtaining optimum design and operating conditions in such devices. Examples of related studies include applications in desiccant dehumidification [1–8], regenerative energy recovery [9–13], as well studies aimed at both cases [14–21]. In dehumidification, regenerators comprise desiccant wheels, having a relatively high sorbent content and operating at slower speeds, whereas in energy recovery enthalpy wheels are used, those of which have a much lower fraction of sorbent and operate at higher speeds.

While most of the previous studies employ formulations that do not include local diffusive resistances to heat and mass transfer in the regenerator's matrix, a few studies [4,5,18,19] have worked with formulations that account for the effects of transversal diffusion. In addition, some studies have included axial diffusion effects in their models [11,18,19], but most formulations apparently do not consider these terms. As shown [22,23], the effects of diffusion in the regenerator's matrix should be considered in many situations, especially those involving enthalpy exchangers. Although previous dimensionless analyses methods have been presented, it has been noted that several studies present rather incomplete normalization schemes, and physically insignificant dimensionless

parameters. The unified formulation presented in [19] employs a reasonable normalization scheme; however, the dimensionless formulation is suitable for particular cases analyzed in that study.

This paper presents a generalized methodology for analyzing heat and mass transfer regenerators, including the effects of heat and mass transfer diffusive resistances in the presence of adsorbent materials. The method extends the ideas presented in well established sensible heat exchanger literature [24] to incorporate the effects of mass transfer, including the effects of adsorption and diffusion of heat and mass in the regenerator's matrix. The potential of the methodology is illustrated through a parametric analysis, in which the influence of the proposed dimensionless groups on the performance of a heat and mass transfer regenerator is investigated.

2. Methodology

The general problem considered in this study is that of an exchanger, which periodically alternates between two different process streams. The streams flow through the numerous exchanger's mini-channels transferring mass and energy to the channel's walls, which consist of porous layers. These layers possess sorbent components, thereby introducing physical adsorption into the problem. The interaction between adjacent channels is negligible, such that heat and mass transport can be reasonably accounted for by focusing on the flow through a single channel. The overall process is adiabatic such that during an entire cycle (composed of two individual processes), mass and energy removed from a

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Nomenclature

A_s	surface area
c, c_p	specific heats
C	sensible heat capacity rate
C_r^*, C^*	sensible heat capacity ratios
Bi	Biot number
\mathcal{D}	mass diffusivity
f_s	sorbent mass fraction
Fi, Fo	Fick and Fourier numbers
h	convective transfer coefficient
i	specific enthalpy
i_{vap}	latent heat of vaporization
i_{sor}	heat of sorption
$i_{v,\Delta T}$	sensible heat of sorbate transfer
k	thermal conductivity
K_f	aspect ratio of porous layer
L	regenerator length
Le	Lewis number
N	number of revolutions
N_{tu}	number of transfer units
ΔR_f	thickness of porous layer
T	temperature
u	bulk stream velocity in channels
V	volumetric capacity rate
V_r^*, V^*	volumetric capacity ratios
W	adsorbed sorbate concentration
Y	gaseous sorbate concentration

Greek symbols

ϵ_f porosity

ϵ	effectiveness
ρ	specific mass
ψ_r	psychrometric ratio
τ	duration of a process
τ_T	period of one revolution
τ_I, τ_{II}	duration of processes I and II
τ_{dw}	dwel time
ζ_g, ζ_s	tortuosities
ϕ	relative humidity

Subscripts

f	porous layer
s	solid phase of porous layer
g	gas-phase of porous layer (pores)
ref	reference value
$p.s.$	at process stream interface
T	entire regenerator

Superscripts

h	sensible heat transfer
m	mass transfer
i	enthalpy transfer
$*$	dimensionless quantity
\star	dry reference quantity

Overscript

\sim	dry basis
$-$	time-averaged value

process stream are entirely delivered to the other one, and vice-versa. Although the solution of each process is transient, after a number of cycles a quasi-steady state is reached, in which these transient solutions repeat themselves for the following cycles. In rotary regenerators a process comprises the period in which a mini-channel travels through a regenerator section. In each of these sections, a different process stream flows through the regenerator's channels.

A formulation for the previously described heat and mass transfer problem was previously developed [19], resulting in equations that account for coupled heat and mass diffusion and convection, as well as adsorption effects. This formulation was shown to be in agreement with experimental results and data from previously published studies. The methodology herein proposed comprises a generalized effectiveness-NTU analysis, which is consistently employed to the dimensional formulation presented in [19].

2.1. Dimensionless groups

Fourier and Fick numbers are defined based on the period of one process (τ):

$$Fo_f = \frac{\alpha_f^* \tau}{\Delta R_f^2}, \quad Fi_g = \frac{\mathcal{D}_g^* \tau}{\Delta R_f^2}, \quad Fi_s = \frac{\mathcal{D}_s^* \tau}{\Delta R_f^2}, \quad (1-3)$$

where two Fick numbers are employed due to gas-phase and surface diffusion in the porous medium. The ratios between Fick and Fourier numbers naturally lead to the definition of Lewis numbers:

$$Le_g = \frac{Fo_f}{Fi_g}, \quad Le_s = \frac{Fo_f}{Fi_s}. \quad (4, 5)$$

The presence of diffusion in the porous felt also requires definitions of Biot numbers:

$$Bi_f^h = \frac{h^h \Delta R_f}{k_f^*}, \quad Bi_g^m = \frac{h^m \Delta R_f}{\mathcal{D}_g^*}, \quad Bi_s^m = \frac{h^m \Delta R_f}{\mathcal{D}_s^*}, \quad (6-8)$$

In order to generalize the effectiveness-NTU method for heat and mass transfer, relevant dimensionless groups are introduced. Numbers of transfer units, for heat and mass transfer, are defined as:

$$N_{tu}^h = \frac{(h^h A_s)}{C}, \quad N_{tu}^m = \frac{(h^m A_s)}{V}, \quad (9, 10)$$

where V is the volumetric flow rate (for an entire regenerator section) and C is the sensible heat capacity rate ($C = \rho^* c_p^* V$). Overall N_{tu} s are also defined:

$$N_{tu,o}^m = \frac{1}{V_{\min}} \left[\frac{1}{(h^m A_s)_{|I}} + \frac{1}{(h^m A_s)_{|II}} \right]^{-1}, \quad (11)$$

$$N_{tu,o}^h = \frac{1}{C_{\min}} \left[\frac{1}{(h^h A_s)_{|I}} + \frac{1}{(h^h A_s)_{|II}} \right]^{-1}. \quad (12)$$

The psychrometric ratio and the dimensionless dwell time are defined as:

$$\psi_r = \frac{N_{tu}^h}{N_{tu}^m} = \frac{N_{tu,o}^h}{N_{tu,o}^m}, \quad \tau_{dw}^* = \frac{L}{u\tau}. \quad (13, 14)$$

The fluid capacity ratios and total matrix capacity ratios are defined for heat and mass transfer:

$$C^* = \frac{C_{\min}}{C_{\max}}, \quad C_r^* = \frac{C_{rf}}{C_{\min}}, \quad (15, 16)$$

$$V^* = \frac{V_{\min}}{V_{\max}}, \quad V_r^* = \frac{V_{rf}}{V_{\min}}. \quad (17, 18)$$

where the matrix capacity rates based on the total mass of the porous solid material in the regenerator and the period of an entire revolution (comprised of both processes):

$$V_r = \frac{(m_f^*)|_{\Gamma}}{\rho_f^* \tau_{\Gamma}}, \quad C_r = \frac{(m_f^* c_f^*)|_{\Gamma}}{\tau_{\Gamma}}. \quad (19)$$

The last set of ε -NTU parameters are the convective resistance ratios:

$$(h^h A_s)^* = \frac{(h^h A_s) \text{ on the } C_{\min} \text{ section}}{(h^h A_s) \text{ on the } C_{\max} \text{ section}}, \quad (20)$$

$$(h^m A_s)^* = \frac{(h^m A_s) \text{ on the } V_{\min} \text{ section}}{(h^m A_s) \text{ on the } V_{\max} \text{ section}}. \quad (21)$$

A last set of dimensionless parameters are also introduced due to the presence of adsorption and coupled heat and mass transfer:

$$i_{sor}^* = \frac{i_{sor} \rho_g^* \Delta Y_{ref}}{\rho_f^* c_f^* \Delta T_{ref}}, \quad (22)$$

$$\Omega = f_s \frac{\rho_s^* W_f^{\max}}{\rho_g^* \Delta Y_{ref}}, \quad (23)$$

$$i_{v,\Delta T}^* = \frac{i_{v,\Delta T} \rho_g^* \Delta Y_{ref}}{\rho_f^* c_f^* \Delta T_{ref}}. \quad (24)$$

Finally, it is important to notice that the parameters N_{tu}^m , N_{tu}^h , Fo_f , Fi_g , Fi_s and τ_{dw}^* can assume different values for process I and process II, i.e. for each regenerator section.

2.2. Dimensionless governing equations

Using the dimensionless groups developed in the previous sections, the transport equations given in [19] are transformed into:

$$(1 - \epsilon_f) \Omega \frac{\partial W_f^*}{\partial t^*} + \epsilon_f \frac{\partial Y_f^*}{\partial t^*} = Fi_s \Omega \nabla_* \cdot (\delta_s \nabla_* W_f^*) + Fi_g \nabla_* \cdot (\delta_g \nabla_* Y_f^*), \quad (25)$$

$$\chi_f \frac{\partial T_f^*}{\partial t^*} = Fo \nabla_* \cdot (\kappa_f \nabla_* T_f^*) + (1 - \epsilon_f) \Omega \left(\frac{\partial W_f^*}{\partial t^*} - Fi_s \frac{\nabla_* \cdot (\delta_s \nabla_* W_f^*)}{1 - \epsilon_f} \right) i_{sor}^*, \quad (26)$$

$$\tau_{dw}^* \frac{\partial Y^*}{\partial t^*} + \frac{\partial Y^*}{\partial x^*} = N_{tu}^m (Y_f^*|_{p.s.} - Y^*), \quad (27)$$

$$\chi \left(\tau_{dw}^* \frac{\partial T^*}{\partial t^*} + \frac{\partial T^*}{\partial x^*} \right) = N_{tu}^h (T_f^*|_{p.s.} - T^*), \quad (28)$$

where the boundary conditions are given by:

$$-\Omega \frac{\delta_s}{Bi_s^m} \frac{\partial W_f^*}{\partial r^*} - \frac{\delta_g}{Bi_g^m} \frac{\partial Y_f^*}{\partial r^*} = (Y^* - Y_f^*), \quad (29)$$

$$-\kappa_f \frac{\partial T_f^*}{\partial r^*} = Bi_f^h (T^* - T_f^*) + \frac{Bi_g^m}{Le_g} (Y^* - Y_f^*) i_{v,\Delta T}^* - \frac{\Omega}{Le_s} \delta_s \frac{\partial W_f^*}{\partial r^*} i_{sor}^*, \quad (30)$$

at $r^* = 0$. At the remaining boundary one finds:

$$\frac{\partial Y_f^*}{\partial r^*} = \frac{\partial T_f^*}{\partial r^*} = 0, \quad \text{at } r^* = 1, \quad (31)$$

$$\frac{\partial Y_f^*}{\partial x^*} = \frac{\partial T_f^*}{\partial x^*} = 0, \quad \text{at } x^* = 0, 1. \quad (32)$$

The periodicity of the problem appears in the inlet conditions:

$$Y^*(0, t^*) = Y_{in}^*(t^*), \quad T^*(0, t^*) = T_{in}^*(t^*), \quad (33, 34)$$

where the inlet quantities have different values in each process stream. Moreover, considering a counterflow arrangement, the change of variable $x_{next}^* = 1 - x_{current}^*$ is applied at the end of each process.

The dimensionless variables in the presented equations are defined by:

$$T^* = \frac{T - T_{ref}}{\Delta T_{ref}}, \quad T_f^* = \frac{T_f - T_{ref}}{\Delta T_{ref}}, \quad (35, 36)$$

$$Y^* = \frac{Y - Y_{ref}}{\Delta Y_{ref}}, \quad Y_f^* = \frac{Y_f - Y_{ref}}{\Delta Y_{ref}}, \quad W_f^* = \frac{W_f}{W_f^{\max}}, \quad (37-39)$$

$$t^* = \frac{t - N(\tau_I + \tau_{II})}{\tau_I}, \quad \text{for process I}, \quad (40)$$

$$t^* = \frac{t - \tau_I - N(\tau_I + \tau_{II})}{\tau_{II}}, \quad \text{for process II}, \quad (41)$$

$$x^* = \frac{x}{L}, \quad r^* = \frac{r - R_p}{\Delta R_f}, \quad \nabla_* = \left(K_f \frac{\partial}{\partial x^*}, \frac{\partial}{\partial r^*} \right), \quad (42-44)$$

where K_f is the aspect ratio $\Delta R_f/L$. The remaining coefficients account for variations in physical properties:

$$\delta_g = \frac{\epsilon_f}{\zeta_g} \frac{\mathcal{D}_g}{\mathcal{D}_g^*}, \quad \delta_s = \frac{1 - \epsilon_f}{\zeta_s} \frac{\mathcal{D}_s}{\mathcal{D}_s^*}, \quad (45, 46)$$

$$\chi = \frac{\rho c_p}{\rho^* c_p^*}, \quad \chi_f = \frac{\rho_f c_f}{\rho_f^* c_f^*}, \quad \kappa_f = \frac{k_f}{k_f^*}, \quad (47-49)$$

3. Results and discussion

After introducing the analysis methodology, simulation results are presented. Solving the obtained formulation by a numerical algorithm based on the Finite Volumes Method and the Method of Lines [25], a parametric analysis is conducted for investigating the effects of the dimensionless parameters on regenerator performance, which is measured in terms of mass and enthalpy effectiveness:

$$\epsilon^m = \frac{V_I}{V_{\min}} \frac{\bar{Y}_{out,I}^* - \bar{Y}_{in,I}^*}{\bar{Y}_{in,II}^* - \bar{Y}_{in,I}^*} = \frac{V_{II}}{V_{\min}} \frac{\bar{Y}_{out,II}^* - \bar{Y}_{in,II}^*}{\bar{Y}_{in,I}^* - \bar{Y}_{in,II}^*}, \quad (50)$$

$$\epsilon^i = \frac{C_I}{C_{\min}} \frac{\bar{i}_{I,out}^* - \bar{i}_{I,in}^*}{\bar{i}_{II,in}^* - \bar{i}_{I,in}^*} = \frac{C_{II}}{C_{\min}} \frac{\bar{i}_{II,out}^* - \bar{i}_{II,in}^*}{\bar{i}_{I,in}^* - \bar{i}_{II,in}^*}, \quad (51)$$

where time-averaged quantities (over the duration of each process) are employed for providing the average inlet and outlet quantities for the entire regenerator sections.

3.1. Relations and common simplifications

According to the definition of the capacity rates the fluid capacity ratios are related to the dimensionless dwell times for each regenerator section through:

$$V^* = C^* = \frac{\tau_{dw}^*|_{C_{\max}}}{\tau_{dw}^*|_{C_{\min}}} = \frac{\tau_{dw}^{\min}}{\tau_{dw}^{\max}}, \quad (52)$$

where $\tau_{dw}^*|_{C_{\max}}$ and $\tau_{dw}^*|_{C_{\min}}$ are the dimensionless dwell times on the C_{\max} and V_{\max} sections. This notation also applies to the rest of this text.

Considering cases for which the convective transfer coefficients (h^h and h^m) are uniform throughout the regenerator, the expressions for the overall numbers of transfer units and the convective

resistance ratios can be simplified, leading to the following relations:

$$N_{tu,o}^h = \frac{(h^h A_s)^*}{1 + (h^h A_s)^*} \frac{N_{tu,h}|_{C_{\max}}}{C^*} = \frac{N_{tu,h}|_{C_{\min}}}{1 + (h^h A_s)^*}, \quad (53)$$

$$N_{tu,o}^m = \frac{(h^m A_s)^*}{1 + (h^m A_s)^*} \frac{N_{tu,m}|_{V_{\max}}}{V^*} = \frac{N_{tu,m}|_{V_{\min}}}{1 + (h^m A_s)^*}. \quad (54)$$

In addition, the expressions for the convective resistance ratios are also simplified, leading to:

$$(h^m A_s)^* = \frac{A_s|_{V_{\min}}}{A_s|_{V_{\max}}} = \frac{A_s|_{C_{\min}}}{A_s|_{C_{\max}}} = (h^h A_s)^*. \quad (55)$$

Since the area ratios are related to the duration of each process, the last equation can be rewritten as:

$$(h^m A_s)^* = (h^h A_s)^* = \frac{\tau|_{V_{\min}}}{\tau|_{V_{\max}}} = \frac{\tau|_{C_{\min}}}{\tau|_{C_{\max}}}. \quad (56)$$

After some manipulation, some of the dimensionless parameters are shown to be related through the following expressions:

$$\psi_r = \frac{N_{tu}^h}{N_{tu}^m} = \frac{N_{tu,o}^h}{N_{tu,o}^m}, \quad (57)$$

$$Fo = K_f^c \frac{N_{tu}^h}{Bi_f^h C_r^*}, \quad (58)$$

$$Fi_g = K_f^c \frac{N_{tu}^m}{Bi_g^m V_r^*}, \quad Fi_s = K_f^c \frac{N_{tu}^m}{Bi_s^m V_r^*}, \quad (59, 60)$$

where the parameter K_f^c is a geometric ratio associated with the curvature of the porous layer. For negligible curvature, which is a reasonable assumption for thin layers, $K_f^c \approx 1$.

If the bulk stream velocity is equal for both processes, the relation between the dimensionless parameters can be further simplified. Under this condition, the following relation is found:

$$C^* = V^* = (h^h A_s)^* = (h^m A_s)^* = \frac{A_s^{\min}}{A_s^{\max}} = \frac{\tau_{\min}}{\tau_{\max}}, \quad (61)$$

$$N_{tu,I}^h = N_{tu,II}^h = (1 + C^*) N_{tu,o}^h, \quad (62)$$

$$N_{tu,I}^m = N_{tu,II}^m = (1 + V^*) N_{tu,o}^m. \quad (63)$$

For cases in which the period of both processes are equal but the bulk stream velocities are allowed to assume different values (i.e. for *symmetric regenerators*), the following relations are obtained:

$$(h^m A_s)^* = (h^h A_s)^* = 1, \quad V^* = C^* = \frac{u_{\min}}{u_{\max}}, \quad (64, 65)$$

$$N_{tu,h}|_{C_{\max}} = C^* N_{tu,h}|_{C_{\min}} = 2C^* N_{tu,o}^h, \quad (66)$$

$$N_{tu,m}|_{V_{\max}} = V^* N_{tu,m}|_{V_{\min}} = 2V^* N_{tu,o}^m. \quad (67)$$

Finally, considering the cases in which the bulk stream velocities and the period of both processes are equal (i.e. for *symmetric and balanced regenerators*), one finds that

$$V^* = C^* = (h^m A_s)^* = (h^h A_s)^* = 1, \quad (68)$$

$$N_{tu,I}^h = N_{tu,II}^h = 2N_{tu,o}^h, \quad (69)$$

$$N_{tu,I}^m = N_{tu,II}^m = 2N_{tu,o}^m. \quad (70)$$

3.2. Analysis of regenerative parameters

Following the proposed ε -NTU approach, effectiveness results, for mass and energy transfer, are presented for various combinations of N_{tu} s and matrix capacity ratios. In order to facilitate comparisons, a base case is considered, and the effect of varying the dimensionless parameters is examined by analyzing the resulting departure from this base case. All results were calculated with grid sizes and temporal integration parameters that ensure at least three significant figures of precision, and the properties needed for evaluating variable coefficients in the equations are given in [Table 8](#).

The base case consists of a symmetric and balanced regenerator (with $(h^h A_s)^* = C^* = 1$), a relatively low storage effect in the process stream ($\tau_{dw}^{\max} = 0.001$), a small porous layer aspect ratio ($K_f = 10^{-3}$), and low diffusional resistances in the porous layer ($Bi_f^h = Bi_g^m = 0.1$). Also, a small sorbent mass fraction is selected ($f_s = 1\%$), together with a linear isotherm ($W_f^* = \phi(T_f, W_f^*)$) and a relatively low heat of sorption ($i_{sor} = 1.2i_{vap}$). In addition, the Lewis approximation, which assumes that $\psi_r = 1$, is considered; as a result, the N_{tu} s for heat and mass transfer have equal values. The results of this case are presented in the first portion of [Table 1](#). Observing the ε^i results, a trend similar to the one observed in sensible heat regenerators [[24,26](#)] is seen, with ε^i increasing for larger C_r^* and N_{tu} . Analyzing the values for the mass effectiveness, a different behavior is observed. For lower matrix capacity ratios, ε^m decreases with increasing N_{tu} s; nevertheless, the values of ε^m are minimal for this range of C_r^* , such that the absolute change in performance is also small. As C_r^* is increased, the mass effectiveness assumes the same behavior observed in the energy effectiveness ε^i .

The next simulations examine the variation of the capacity ratio C^* , and the departure from the base case is pictured in the remaining portions of [Table 1](#), for $C^* = 0.7$ and $C^* = 0.5$. As observed, the performance generally increases as the capacity ratio is decreased. For the mass transfer, it is interesting to note that for the same range of C_r^* (where a decrease with N_{tu} s was observed), the values of ε^m also decrease with C^* . For larger matrix capacity ratios, the mass effectiveness resumes the general behavior, increasing as C^* increases.

Subsequently, variations in other parameters related to regenerative exchange are investigated, starting with increasing the dimensionless dwell time. [Table 2](#) presents performance values for cases with $\tau_{dw}^{\max} = 0.01$ and 0.1 , showing that the effectiveness, for both heat and mass, increases as τ_{dw}^{\max} . This variation in effectiveness is apparently equal for both mass and enthalpy transfer; nevertheless, it can be observed that the increase in ε^m and ε^i is much more pronounced for lower values of C_r^* . The increase in effectiveness for higher τ_{dw}^{\max} is related to the larger amount of fluid carried-over from one process to the other. This increases the calculated values of ε^m and ε^i because the average outlet temperature and concentration from process I will be closer to the inlet potentials of process II.

In sensible heat regenerators, it is known that for values of $(h^h A_s)^*$ between $1/4$ and 4 , there is little change in effectiveness [[24,26](#)]. In order to verify whether this is also true for heat and mass transfer regenerators, changes from the base case for $(h^h A_s)^* = 1/4$ and $(h^h A_s)^* = 4$ are presented in [Table 3](#). As can be seen, for $(h^h A_s)^* = 4$ there is a slight decrease in ε^m and ε^i for $C_r^* = 1$, and for the remaining values of C_r^* a very small increase is observed. For the case with $(h^h A_s)^* = 1/4$ a general, but minimal, decrease in effectiveness is observed. Since the departure from the case with $(h^h A_s)^* = 1$ is minor, one could state that the performance results are, in practice, nearly unaffected by the variation of $(h^h A_s)^*$ between $1/4$ and 4 .

Lastly, as displayed in [Table 4](#), the impact caused by departing from the conventional psychrometric ratio (around unity) is

Table 1
Effectiveness results for base case and C_r^* variations.

$N_{tu,o}^h$	$C_r^* = 1$		$C_r^* = 2$		$C_r^* = 5$		$C_r^* = 10$	
	ε^m	ε^i	ε^m	ε^i	ε^m	ε^i	ε^m	ε^i
<i>Base case</i>								
1	0.1304	0.2533	0.2754	0.3539	0.4024	0.4384	0.4334	0.4584
2	0.1106	0.2927	0.2913	0.4210	0.5256	0.5770	0.5885	0.6180
3	0.0998	0.3120	0.2898	0.4485	0.5873	0.6462	0.6706	0.7005
5	0.0885	0.3322	0.2844	0.4736	0.6533	0.7180	0.7583	0.7864
10	0.0777	0.3525	0.2791	0.4962	0.7240	0.7900	0.8479	0.8705
50	0.0701	0.3750	0.2917	0.5251	0.8262	0.8802	0.9551	0.9634
$C_r^* = 0.7$								
1	0.1228	0.2609	0.2712	0.3648	0.4203	0.4638	0.4598	0.4892
2	0.0989	0.3061	0.2737	0.4340	0.5446	0.6134	0.6333	0.6707
3	0.0858	0.3287	0.2644	0.4629	0.6001	0.6842	0.7243	0.7640
5	0.0719	0.3525	0.2494	0.4886	0.6494	0.7498	0.8189	0.8574
10	0.0576	0.3765	0.2297	0.5076	0.6813	0.7967	0.9082	0.9376
50	0.0409	0.4018	0.1996	0.5083	0.6723	0.8005	0.9852	0.9913
$C_r^* = 0.5$								
1	0.1178	0.2662	0.2672	0.3717	0.4314	0.4806	0.4776	0.5102
2	0.0919	0.3147	0.2616	0.4421	0.5518	0.6338	0.6607	0.7037
3	0.0784	0.3385	0.2488	0.4713	0.5992	0.7013	0.7535	0.7993
5	0.0647	0.3624	0.2311	0.4955	0.6349	0.7569	0.8444	0.8878
10	0.0521	0.3843	0.2118	0.5089	0.6535	0.7873	0.9221	0.9519
50	0.0401	0.4025	0.1947	0.5055	0.6586	0.7921	0.9842	0.9908

Table 2
Effect of varying τ_{dw}^{\max} .

$N_{tu,o}^h$	$C_r^* = 1$		$C_r^* = 2$		$C_r^* = 5$		$C_r^* = 10$	
	ε^m	ε^i	ε^m	ε^i	ε^m	ε^i	ε^m	ε^i
$\tau_{dw}^{\max} = 0.010$								
1	0.1400	0.2610	0.2840	0.3605	0.4080	0.4430	0.4377	0.4620
2	0.1204	0.3003	0.3006	0.4279	0.5311	0.5810	0.5919	0.6206
3	0.1097	0.3195	0.2995	0.4554	0.5928	0.6501	0.6735	0.7026
5	0.0985	0.3396	0.2944	0.4805	0.6589	0.7218	0.7606	0.7880
10	0.0878	0.3598	0.2893	0.5030	0.7298	0.7938	0.8496	0.8716
$\tau_{dw}^{\max} = 0.100$								
1	0.2358	0.3375	0.3679	0.4261	0.4633	0.4878	0.4801	0.4978
2	0.2187	0.3755	0.3932	0.4952	0.5840	0.6201	0.6248	0.6459
3	0.2087	0.3938	0.3960	0.5235	0.6451	0.6870	0.7016	0.7232
5	0.1984	0.4127	0.3939	0.5490	0.7108	0.7571	0.7831	0.8037
10	0.1888	0.4314	0.3907	0.5714	0.7820	0.8283	0.8653	0.8822

Table 3
Effect of varying $(h^h A_s)^*$.

$N_{tu,o}^h$	$C_r^* = 1$		$C_r^* = 2$		$C_r^* = 5$		$C_r^* = 10$	
	ε^m	ε^i	ε^m	ε^i	ε^m	ε^i	ε^m	ε^i
$(h^h A_s)^* = 1/4$								
1	0.1171	0.2481	0.2448	0.3366	0.3906	0.4316	0.4303	0.4566
2	0.1039	0.2921	0.2656	0.4076	0.5096	0.5680	0.5850	0.6161
3	0.0950	0.3126	0.2706	0.4391	0.5699	0.6365	0.6672	0.6987
5	0.0852	0.3334	0.2728	0.4687	0.6349	0.7076	0.7554	0.7848
10	0.0757	0.3538	0.2741	0.4948	0.7048	0.7788	0.8459	0.8693
$(h^h A_s)^* = 4$								
1	0.1399	0.2560	0.2910	0.3613	0.4096	0.4424	0.4364	0.4600
2	0.1162	0.2933	0.3080	0.4286	0.5389	0.5844	0.5928	0.6204
3	0.1038	0.3119	0.3035	0.4543	0.6030	0.6549	0.6751	0.7030
5	0.0914	0.3316	0.2932	0.4766	0.6703	0.7275	0.7621	0.7884
10	0.0795	0.3515	0.2832	0.4969	0.7412	0.7998	0.8500	0.8715

analyzed. The value of $N_{tu,o}^h$ is still varied within the same ranges, but, due to having $\psi_r \neq 1$, the values for $N_{tu,o}^m$ are now twice the values of $N_{tu,o}^h$ for $\psi_r = 0.5$ and half for case with $\psi_r = 2$. The results reveal that, with $\psi_r = 0.5$ and small values of $C_r^* = 1$, there is a general, yet small, decrease in ε^m and ε^i .

In contrast, the larger values of C_r^* lead to a noticeable increase in ε^m and ε^i . With $\psi_r = 2$, the opposite trend is observed. These results show how adopting an imprecise value for the psychrometric ratio can result in a poor estimate for exchanger performance.

Table 4
Effect of varying ψ_r .

$N_{tu,o}^h$	$C_r^* = 1$		$C_r^* = 2$		$C_r^* = 5$		$C_r^* = 10$	
	ε^m	ε^i	ε^m	ε^i	ε^m	ε^i	ε^m	ε^i
$\psi_r = 0.5$								
1	0.1281	0.2482	0.3057	0.3627	0.5308	0.5026	0.5901	0.5390
2	0.1077	0.2893	0.2985	0.4185	0.6307	0.6292	0.7238	0.6876
3	0.0974	0.3097	0.2909	0.4439	0.6772	0.6911	0.7861	0.7602
5	0.0868	0.3308	0.2828	0.4689	0.7251	0.7547	0.8483	0.8334
10	0.0766	0.3519	0.2775	0.4931	0.7746	0.8168	0.9080	0.9025
$\psi_r = 2$								
1	0.1261	0.2585	0.2168	0.3313	0.2716	0.3728	0.2870	0.3829
2	0.1143	0.2991	0.2639	0.4162	0.3990	0.5147	0.4327	0.5381
3	0.1038	0.3170	0.2767	0.4505	0.4728	0.5901	0.5242	0.6254
5	0.0918	0.3353	0.2816	0.4792	0.5569	0.6703	0.6337	0.7222
10	0.0797	0.3540	0.2805	0.5014	0.6513	0.7528	0.7575	0.8232

3.3. Analysis of diffusion parameters

In this next series of simulations, the effects of diffusional resistances is investigated. Since the usual values of surface diffusivity are significantly smaller than the diffusivity in the gas-phase of the pores, Bi_s^s is considerably larger than Bi_g^m . For the considered temperature range, the ratio between mass diffusivities $Bi_f^{m*} = Bi_g^m / Bi_s^s$ equals 10^{-5} , and this value is maintained for all simulated cases.

Initially, the effect of separately varying heat and mass transfer Biot numbers is analyzed. Table 5 displays the effects of varying Bi_g^m for a low heat conduction resistance ($Bi_f^h = 0.1$). As one can observe, in general, the effectiveness values for both heat and mass transfer decrease at a notable rate as Bi_g^m is increased. Also, it is interesting to note that in the region where ε^m decreases with N_{tu} , increasing the resistance to mass diffusion leads to the opposite effect. It can also be observed that the change in the effectiveness values is more pronounced for higher values of C_r^* . In addition, it should be said that the change in the effectiveness values as Bi_g^m is increased has a much greater effect on ε^m than on ε^i .

The next set of simulations investigate the effect of varying the heat conduction resistance in the felt. Table 6 presents the differences in effectiveness obtained from using different values of Bi_f^h . As can be seen, increasing Bi_f^h has a minuscule effect on the mass effectiveness, producing a small increase in ε^m for lower C_r^* values and a minor decrease for higher values of C_r^* , with the changes being slightly higher for low C_r^* values. On the enthalpy effectiveness, varying Bi_f^h has a much more visible effect, degrading the values of ε^i as Bi_f^h is raised. However this variation is clearly more pronounced for the lower values of $N_{tu,o}^h$. After comparing the effects of varying the heat and mass transfer Biot numbers, it is clear that Bi_g^m has a greater effect on effectiveness while compared to Bi_f^h .

Table 5
Effect of varying Bi_g^m for $Bi_f^h = 0.1$.

$N_{tu,o}^h$	$C_r^* = 1$		$C_r^* = 2$		$C_r^* = 5$		$C_r^* = 10$	
	ε^m	ε^i	ε^m	ε^i	ε^m	ε^i	ε^m	ε^i
$Bi_g^m = 1$								
1	0.1129	0.2565	0.1778	0.3123	0.2520	0.3604	0.3094	0.3923
2	0.1132	0.3033	0.2319	0.4043	0.3455	0.4858	0.4182	0.5283
3	0.1057	0.3214	0.2552	0.4450	0.4101	0.5574	0.4872	0.6044
5	0.0944	0.3385	0.2718	0.4796	0.4946	0.6386	0.5793	0.6929
10	0.0815	0.3556	0.2788	0.5045	0.5991	0.7262	0.7029	0.7943
$Bi_g^m = 10$								
1	0.0444	0.2344	0.0729	0.2630	0.1263	0.2959	0.1901	0.3300
2	0.0549	0.2980	0.0950	0.3462	0.1646	0.3936	0.2384	0.4345
3	0.0635	0.3291	0.1111	0.3895	0.1929	0.4472	0.2745	0.4934
5	0.0765	0.3598	0.1369	0.4368	0.2352	0.5078	0.3280	0.5615
10	0.0890	0.3820	0.1842	0.4890	0.3053	0.5788	0.4132	0.6425

The last set of results, presented in Table 7, illustrates the variation of both heat and mass transfer resistances. The same trend seen for separately increasing both Biot numbers is again observed, but the combined effect leads to greater change in the effectiveness values. The values of ε^m and ε^i generally decrease with increasing Bi_f^h and Bi_g^m , except for low C_r^* , where the mass transfer effectiveness increases as Bi_f^h is increased to one. However, as it is seen, for $Bi_f^h = 10$ the phenomena where ε^m decreases with $N_{tu,o}^h$ is reversed to the usually expected behavior.

These results demonstrate how heat and mass diffusion resistances can play an important role in exchanger performance, especially with regards to mass transfer. The indication that Biot numbers equal to one generate greater change in the effectiveness for larger C_r^* values suggests that the importance of diffusion resistances will be greater for faster rotation (e.g. enthalpy wheels), since C_r^* is directly proportional to rotational speed. However, for larger Biot numbers, a larger range of C_r^* is affected and diffusion effects can also become relevant for slower rotation (e.g. desiccant wheels).

4. Summary and conclusions

This paper provided an extension of the effectiveness-NTU methodology for regenerator analysis, covering cases with coupled heat and mass transfer, physical adsorption, and diffusional resistances in the matrix. A parametric analysis of a heat and mass transfer regenerator was carried out with the proposed methodology, illustrating its potential and demonstrating the specific influence that different parameters can have on regenerator performance. The results provide insight on ways for improving performance and show how can inappropriate choices for dimen-

Table 6
Effect of varying Bi_f^h for $Bi_g^m = 0.1$.

$N_{tu,o}^h$	$C_r^* = 1$		$C_r^* = 2$		$C_r^* = 5$		$C_r^* = 10$	
	ε^m	ε^i	ε^m	ε^i	ε^m	ε^i	ε^m	ε^i
$Bi_f^h = 1$								
1	0.1362	0.2361	0.2786	0.3328	0.4001	0.4175	0.4282	0.4410
2	0.1165	0.2777	0.2957	0.4018	0.5256	0.5547	0.5855	0.5980
3	0.1051	0.2992	0.2938	0.4320	0.5880	0.6260	0.6689	0.6813
5	0.0928	0.3221	0.2871	0.4608	0.6541	0.7019	0.7576	0.7703
10	0.0804	0.3457	0.2796	0.4877	0.7241	0.7796	0.8476	0.8600
$Bi_f^h = 10$								
1	0.1508	0.1822	0.2817	0.2809	0.3954	0.3817	0.4237	0.4220
2	0.1396	0.2119	0.3061	0.3348	0.5212	0.5024	0.5800	0.5655
3	0.1306	0.2327	0.3075	0.3608	0.5844	0.5675	0.6635	0.6435
5	0.1177	0.2616	0.3022	0.3917	0.6514	0.6404	0.7529	0.7294
10	0.1002	0.2994	0.2904	0.4326	0.7219	0.7217	0.8445	0.8203

Table 7
Effect of varying Bi_g^m and Bi_f^h .

$N_{tu,o}^h$	$C_r^* = 1$		$C_r^* = 2$		$C_r^* = 5$		$C_r^* = 10$	
	ε^m	ε^i	ε^m	ε^i	ε^m	ε^i	ε^m	ε^i
$Bi_g^m = Bi_f^h = 1$								
1	0.1170	0.2376	0.1775	0.2872	0.2384	0.3281	0.2888	0.3613
2	0.1182	0.2877	0.2346	0.3830	0.3400	0.4573	0.3996	0.4935
3	0.1104	0.3082	0.2584	0.4272	0.4078	0.5337	0.4748	0.5745
5	0.0982	0.3283	0.2745	0.4663	0.4943	0.6212	0.5738	0.6715
10	0.0841	0.3488	0.2797	0.4960	0.5996	0.7159	0.7012	0.7822
$Bi_g^m = Bi_f^h = 10$								
1	0.0368	0.1297	0.0544	0.1619	0.0929	0.2116	0.1431	0.2567
2	0.0550	0.1838	0.0768	0.2210	0.1274	0.2845	0.1872	0.3394
3	0.0696	0.2225	0.0966	0.2629	0.1543	0.3306	0.2221	0.3911
5	0.0883	0.2712	0.1307	0.3242	0.1973	0.3913	0.2756	0.4574
10	0.1025	0.3205	0.1873	0.4096	0.2798	0.4842	0.3643	0.5466

Table 8
Input data used in simulations.

Specific mass of dry air	1.1614 kg/m ³
Specific heat of dry air	1007 J/kg °C
Thermal conductivity of dry air	26.3 mW/m °C
Specific heat of water vapor	1872 J/kg °C
Thermal conductivity of water vapor	19.6 mW/m °C
Specific heat of saturated liquid water	4180 J/kg °C
Thermal conductivity of saturated liquid water	613 mW/m °C
Average gas-phase mass diffusivity (\mathcal{D}_g^*)	25 mm ² /s
Average surface mass diffusivity (\mathcal{D}_s^*)	0.00025 mm ² /s
Specific mass of dry matrix	800 kg/m ³
Specific heat of dry matrix	920 J/kg °C
Thermal conductivity of dry porous layer	240 mW/m °C
Equilibrium relation	$W_f^* = \phi(T_f^*, Y_f^*)$
Heat of sorption	$i_{sor} = 1.2i_{vap}$
Tortuosities (ζ_g, ζ_s)	3.0
Stream I inlet temperature	30 °C
Stream II inlet temperature	15 °C
Stream I inlet relative humidity	40%
Stream II inlet relative humidity	40%

sionless parameters affect the predicted effectiveness results. The results of varying diffusional resistances show how these can play an important role in exchanger performance, indicating a clear need for future research.

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